Chapter 2
The Measurement of the Structure of the National Economy Major Variables in Macroeconomy

Introduction
How we measure the variables that we consider to be part of the structure is imp. We will go into conceptual and practical issues with these variables. Use the National Income and Product Accounts (NIPA). Organized in a way that helps understand economic structure.

Economic Activity is measured over time – i.e., it measures a flow

Circular Flow Diagram: 3 equivalent ways to measure economic activity.
1. Product (how much produced at each stage of production)
2. Income (how much income received by producers at each stage)
3. Expenditure (amount spent at each stage)

GDP: The Market value of all final goods and services produced within the borders of a nation over some period of time (usually a year, although preliminary figures may be released quarterly.

1. What is it used for?
   a. Measure of our standard of living (GDP per capita)
   b. Measure of economic activity

2. Limitations: Exclusions and Inclusions that do not make this a good measure of the above
   a. Underground economy
   b. Some types of self employment
   c. Public services are measured at cost
   d. Disasters and the environment (NIPA attach no value to a clean environment. Implies that production that pollutes adds the value of production to our GDP. Disaster clean-up)
   e. Capital goods (not final, but not used up. Assume the producer purchases what is left). We do not consume and therefore not part of standard of living.

3. GDP vs. GNP
   a. GNP: market value of G&S produced by domestically owned inputs during a year
   b. GDP = GNP – NFP
      Net Factor Payments (NFP) receipts from our inputs that are in foreign countries – payments that foreign inputs in our country make to their country. NFP is currently negative for the US (i.e. foreign inputs collect more from the US than we collect on our inputs abroad). This is why GDP > GNP for the US.
   c. Example of GNP, GDP and NFP
      GNP = $10
Payments made to the US from our inputs abroad = $2
Payments to Foreigners for their inputs in our country = $3
\[ \text{GDP} = \text{GNP} - \text{NFP} \]
\[ 10 = \text{GNP} - (2 - 3) \]
\[ \text{GNP} = 9 \]

**The Expenditure Approach to Measuring GDP** – Who buys it? Eventually we will use each of these variables to the Microeconomic Behavior behind demand for goods and services (i.e., Aggregate Demand).

Already established that \( Y = \text{GDP} = \text{Income} = \text{Expenditure} = 11.75 \) trillion (2005)

Now we will decompose Expenditures

\[ \text{GDP} (Y) = C + I + G + NX, \quad (NX = X - M) \]

**Components of GDP and their share of GDP averaged from 1974-2004**

1. **C**: 66% of GDP (range: 62.1-70.5%; Std. Dev.: 2.5%)
   a. Durables (goods that typically last for at least 3 years: e.g., TV, Car and Washing Machines)
   b. Non-durables (goods that typically last for less than 3 years: e.g., food and clothing)
   c. Services (healthcare, haircuts, transportation, insurance, financial services)
   d. Property of Consumption Expenditures: Large and Stable

2. **I**: 16% of GDP (range: 13.4-19.2%; Std. Dev.: 1.5%)
   a. Fixed I
      (1) Business fixed investment (structures and equipment)
      (2) Residential housing
   b. Inventories
   c. Other and services
   d. Property of I: small, but very volatile

3. **G**: 19\% of GDP
   a. Purchases (at cost) by federal and Sate & local governments.
   b. Transfers (welfare, Medicare, Social Security, unemployment benefits, and interest on the national debt (15\% of government expenditures)

4. **Net Exports** (\( NX = X - M \)): -1\%
   a. Exports - value of what is sold to other countries
   b. Imports - value of what is purchased from other countries. This is added to C, I and G and then subtracted from these in M.
Disposable Income – Key Component of Consumption and will use to derive the sources and uses of private savings

$DI = Y + NFP + TR + INT \text{ (from govt. debt)} - T$

Where, $Y = GDP$ (and $Y + NFP = GNP$)

$TR = \text{transfers from the government to the private sector. This is composed of Social Security Payments, Medicare, Unemployment Insurance, Welfare, etc.}$

$INT = \text{Interest on the National Debt (now about 15\% of total govt. expenditures)}$

This is paid to people so is therefore part of their income.

$T = \text{taxes. These are subtracted from income.}$

Disposable Income - $DI$ (or $Y_d$) For simplicity we can combine terms

$DI = (Y + NFP) - (T - TR - INT)$

($GNP$ (net government income))

$DI$ is important because it is thought to be the major determinant of Consumption Expenditures and Savings.

Equilibrium in the Macroeconomy – Sources and Uses of Savings

Small Digression on Stocks and Flows

1. Savings is a flow (measured over time)
2. Wealth is a stock (measured at one point in time). It is the accumulation of savings - so they are related, i.e. $S = \Delta W$. Besides income, wealth is a measure of a standard of living. Wealth = Assets - Liabilities (technically)

National Savings

1. Private Savings ($S_{pvrt}$) Savings by Households and Businesses

$S_{pvrt} = DI - C$

$= Y + NFP + TR + INT - C - T$

2. Government Savings ($S_{govt}$)

$S_{govt} = T - (G + TR + INT)$, savings (or surplus) if $> 0$ (or T-G)

deficit if $< 0$ (or G-T>0).

3. National Savings

$S = S_{pvrt} + S_{govt} = (Y + NFP - T + TR + INT - C) + (T - TR - INT - G)$

$= Y + NFP - C - G$

$= GNP - C - G$ (can see that increasing $C$ and/or $G$ with Taxes and Income constant will decrease National Savings)
Implications – Sources and Uses of Savings (Simple Analysis of how the Macroeconomy Works)

\[ S = Y + NFP - C - G \]
\[ = C + I + G + NX + NFP - C - G \]
\[ = I + (NX + NFP) \]

(Current Account Balance - payments received from abroad in exchange for G & S minus payments made to foreigners)

\[ = I + CA \]

(CA includes imports, exports and factor payments. Ownership abroad affects CA).

In US now, NFP < 0, contributing to a CA deficit. If Foreigners pay us more than we pay them, this increases our savings.

Sources and Uses of Savings Under Various Assumptions about Deficits:

1. When there is a Government Deficit and Trade (CA) surplus
   \[ S_{pvt} = I + (-S_{govt}) + CA \]

2. When there is a Government Surplus and Trade (CA) surplus
   \[ S_{pvt} + S_{govt} = I + CA \]

3. When there is a Government budget deficit and trade deficit
   \[ S_{pvt} - CA = I + (-S_{govt}) \]

4. When there is a Government Surplus and Trade Deficit
   \[ S_{pvt} - CA + S_{govt} = I \]

5. Concepts that follow from the above:
   a. Crowding Out of Private Investment
   b. Twin Deficits – Government Deficit causes Trade Deficit (see Ch. 5)
   c. Productivity of Investment (what the savings goes into) is what matters for whether deficits are Good or Bad (not the Deficit itself).
**Price (Indexes) and Inflation**

**Importance of Prices**
1. Separate real GDP from Prices (i.e. separate nominal GDP into real and price components)
2. Measure the Cost of Living (COLA and COLI)
3. Measure inflation, which is the deterioration in the value of money and affects the efficiency of decisions and causes distributional effects.

**Price Indexes**
(Index number - a summary number – summarizes how the prices of a group of goods and services are changing)

1. Example of the concept of an index number.

   Price of a Hamburger in 1992 = $1.50  
   Price of a Hamburger in 1996 = $2.25  

   Price Index \(_{(\text{Hamburgers 1996})}\)  = $2.25/$1.50  (devoid of $ prices)

   This index is a comparison of prices between a given year (base year) and any other year.

   Price Index \(_{(\text{Hamburgers 1992})}\)  = $1.50/$1.50

2. Types of Price Indexes
   a. GDP Deflator
   b. CPI
   c. PPI
   d. WPI

3. Problems with Indexes (they summarize information to simplify analysis, but they have several short-comings)
   a. Fixed basket indexes ignores substitution => overstates the true cost of living
   b. Does not perfectly capture changes in quality (for all indexes - frequent base year changes help to rectify this)

**Inflation**

Defn. \( \pi = \frac{\% \Delta \text{(Price Index)}}{\text{(Price Index)}_{\text{base year}}} = \frac{\text{(Price Index)}_{\text{current year}} - \text{(Price Index)}_{\text{base year}}}{\text{(Price Index)}_{\text{base year}}} \)
Separating Nominal GDP into Real GDP and the GDP Price Deflator

1. GDP = (Price Index)*(Real GDP)

Separating-out the latter is important for measuring real progress in terms of our standard of living.

2. Real GDP (Concept and Measurement)

   (1) recall that a price index is expressed as  \( \frac{P_{1996}}{P_{1992}} \)

   (2) Nominal GDP is expressed as  \( (P_{1996}Y_{1996}) \)

   (3) Real GDP is calculated as,  \( (Real \ GDP)_{1996} = \frac{(P_{1996}Y_{1996})}{(P_{1996}/P_{1992})} \)

   \[ = \frac{P_{1992}Y_{1996}}{(P_{1992}/P_{1992})} \]

   This can be compared with  \( (Real \ GDP)_{1992} = (P_{1992}Y_{1992}) \)

Then, real GDP growth between 1992-1996 is  \( \frac{(P_{1992}Y_{1996}) - (P_{1992}Y_{1992})}{(P_{1992}Y_{1992})} \)

**Interest Rates – Nominal and Real**

1. Real Interest Rates (\( r \)) = Premium for giving up current consumption

   (In an inflationless world - this is actually expected, where the expectation is for receiving the funds for the lender - i.e. taking into account the probability of default, since a loan is a promise to pay and promises are sometimes broken).

2. Expected Inflation Rate: In a World of Inflation, the lender is paid in money that deteriorates at the rate of inflation. Inflation is not known for sure so borrowers and lenders must form expectations of inflation and form a loan contract based on these expectations.

   \( (Expected \ inflation) \ \pi^e \)

3. Nominal Interest Rate:  \( r + \pi^e \)

4. Decisions to Lend and Borrow based on: Expected Real Interest Rate (\( r^e \))
Chapter 3 The Determination of Output (Labor and Labor Productivity: The Building Blocks of Aggregate Supply)

Introduction

1. The amount of output produced by an economy depends on its physical capacity to produce goods, i.e. the
   a. Quantity of inputs
   b. Productivity of inputs

2. The input with the largest share of total cost (and therefore, productivity) is labor

3. This chapter we look at
   a. The determinants of the quantity and price of labor
   b. Express labor output relationship in terms of unemployment

Production Function – Theoretical Relationship between Inputs and Output

\[ Y = AF(K, L), \]

*Short-Run Properties of the Production Function (these are technical characteristics of productivity, given input quality).* “A” represents input factor quality.

(1) \[ \frac{dY}{dL} > 0 \text{ and } \frac{d}{dL}\left(\frac{dY}{dL}\right) < 0 \text{ (i.e. diminishing returns to labor, assuming capital is fixed)} \]

and where

(2) \[ \frac{dY}{dK} > 0 \text{ and } \frac{d}{dK}\left(\frac{dY}{dK}\right) < 0 \text{ (i.e. diminishing returns capital, assuming labor is fixed)} \]

*Long-Run Properties of the Production Function:*

(1) If \( 2K > 2L \), Y more than doubles increasing returns to scale (exponents on L & K sum to > 1)

(2) If \( 2K > 2L \), Y less than doubles decreasing returns to scale (exponents on L & K sum to < 1)

(3) If \( 2K = 2L \), Y doubles constant returns to scale (exponents on L and K sum to 1)

Note:

a. (1) and (2) are short-run properties of the production function (i.e. when at least one of the inputs is constant).

b. (3) is a long-run property of the production function (i.e. when all inputs are variable).

c. \( \frac{dY}{dL} \) and \( \frac{dY}{dK} \) are the marginal productivity of labor (MPL – slope of the production function) and capital (MPK – slope of the production function) respectively, which are both positive (1st derivative), but declining (2nd derivative).

Assuming a production function of the form:

\[ Y = AK^\alpha L^\beta \]

This function can be estimated using data by linear regression after taking the logarithms.

\[ \ln(Y) = \ln(A) + \alpha \ln(K) + \beta \ln(L) + \varepsilon \]

where \( \alpha = 0.3 \)
\( \beta = 0.7 \)

*Can be used to compute "A" (TFP) given values of Y, K and L.

2. Graphically, MPL

3. Shifts in the Production Function are affected by
   a. \( \Delta \) fixed input
   b. \( \Delta \) Labor Productivity
   c. \( \Delta \) Technology

Shifts in the above production function cause changes in MPL and MPK for a given quantity of L.
**Demand for Labor** - Labor hiring and output decision of the firm

1. Assumptions
   a. Homogeneous labor
   b. Capital is constant => diminishing MPL as labor increases
   c. Competitive output market => output price (p) given to the firm
   d. Competitive labor market => nominal wage rate (w) is given to the firm
   e. Firms wish to maximize profits => they set MR = MC

The profit maximizing condition in terms of the labor hiring decision is,

\[ p_{MPL} = w, \]

i.e. the firm will hire more labor as long as \( p_{MPL} > w \) and will stop hiring when (4) is met.

Eq. (4) can be expressed in real terms as,

\[ MPL = \frac{w}{p} \]

(i.e. the real wage rate \( \frac{w}{p} \) is equal to the MPL of the last laborer hired, or the firm should not hire past the point where MPL is less than the real wage (in terms of G&S) paid to the worker. Since \( w \) and \( p \) are determined in the output and labor markets and not by the firm, the firm can only make a hiring decision. This decision determines the number of laborers and the level of output. Given \( \frac{w}{p} \), their demand for labor depends only on the productivity of labor.

\( \frac{w}{p} \) is the price of labor. As with the derivation of any demand function, price is given and we map-out the quantity response of the agent.

Graphically, this labor hiring decision is,

![Graphical representation of labor hiring decision](image)

Where \( L^D \) is MPL, which is set equal to \( \frac{w}{p}_0 \) which determines \( L^* \) (the optimal level of labor). Through the production function \( L^* \) determines the optimal level of output by the firm.

Shifts in \( L^D \) (relationship with)
   a. \( \Delta \) capital (+)
   b. \( \Delta \) Labor Productivity (+)
   c. \( \Delta \) Technology (+)
**Labor Supply**
This decision involves a tradeoff between work (income) and leisure. As real wages rise the quantity supplied of labor rises. I.e. laborers are willing to give up leisure for income (but only to a point, then $L^S$ is backward bending).

Factors that shift $L^S$
- a. Wealth (-)
- b. expected future real wage (-)
- c. working-age population (+)
- d. participation rate (+)

**Labor Market Equilibrium**
1. Where $L^D = L^S$ the labor market is in equilibrium. This determines the full-employment level of labor and the equilibrium real wage rate.
2. Shocks – factors that shift labor supply and demand. These are determined outside the labor market, but affect the equilibrium quantity of labor and the real wage.
   - e.g. a. An increase in the MPL will shift $L^D$ to the right and increase employment and the real wage. (try some of these shocks yourself to determine the new equilibrium level of employment and real wage rate).
   - b. what effect will a higher minimum real wage have?
   - c. Try double shifts.

   [See article on minimum wage and its effects on employment]

**Unemployment**
1. Definition $U = \frac{\# \text{ of unemployed and actively seeking work}}{\text{labour force}}$, where labor force = $\# \text{ of working} + \# \text{ unemployed and actively seeking work}$
2. Discouraged worker = unemployed and no longer actively seeking employment

[See article on two ways of measuring unemployment]
**Types of Unemployment** - Policies to mitigate them

a. **Frictional** = unemployment associated with being between jobs (voluntary). Useful for our economy and personally. Affected by search-time needed to match workers with vacancies. Will be reduced with growth of job registries.

b. **Structural** = unemployment associated with a structural change in the economy (e.g. a dying industry) (not voluntary). Affected by people changing preferences and foreign competition. Will be reduced with job training programs.

c. **Cyclical** = unemployment associated with the general expansions and contractions in the aggregate economy. Reduced with Fiscal and Monetary policies.

(U*) **Natural Rate of Unemployment**: If only Frictional and Structural unemployment exist (we are considered fully employed). Alternative Definition that we will refer to later - a balance between job seekers and openings, such that nominal wages neither increase or decrease.

**Okun’s Law – The Relationship Between Unemployment and Real GDP Growth**

\[
\frac{(Y^* - Y)}{Y^*} = 2.0 \left( U - U^* \right)
\]

where \( Y^* \) is the full-employment level of real GDP and \( U^* \) is the Natural Rate of Unemployment

Interpretation: If \( U \) increases (decreases) from \( U^* \) by 1\%, Real GDP growth will decrease (increase) by 2\%. Alternatively, when \( U \) increases by 1\% above \( U^* \), \( Y \) falls 2\% below the long-run (or potential) GDP growth rate.
Chapter 4
Consumption (C), Savings (S), and Investment (I)
(The Building Blocks of Aggregate Demand)

Introduction

1. Chapter 3 dealt with factors that determine the supply of output
2. Chapter 4 concerns factors that affect the AD for Goods & Services by consumers and businesses.
   a. S and I determine K formation and thus determine L-run growth in the economy
   b. S-run changes in S & I (and changes in their determinants) affect S-run fluctuations in output
      and thus play a role in business cycles.

Desired Consumption and Savings Expenditures (C^d & S^d)

1. Features of C and S
   a. C is the largest component of GDP (about 2/3)
   b. C is linked to S. A decision to C is also a decision to S.
   c. C and S depend on the tradeoff between current and future consumption. More C currently
      means less S and therefore less future C.

2. Determinants of C^d (desired consumption) and S^d (desired savings)
   a. Current Income (Y)
      (1) \( \Delta Y \uparrow \Rightarrow \Delta C^d \uparrow \Rightarrow \Delta S^d \uparrow \)
      (2) Assuming a linear relationship between Y and C^d
      \[ C^d = C_A + c_Y Y \] (consumption function)
      where, \( \Delta C^d/\Delta Y = c_Y \)
      and where, \( 0 < c_Y < 1 \) (\( c_Y \) is the marginal propensity to consume (MPC))
      (3) From the above,
      \[ S^d = Y - C^d = -C_A - c_Y Y + Y \]
      where, \( 0 < (1 - c_Y) < 1 \) ((1 - c_Y) is the marginal propensity to save (MPS))
      and \( MPC + MPS = 1 \)
   b. Expected Future Income (Y^e)
      (1) \( \Delta Y^e \uparrow \Rightarrow \Delta C^d \uparrow \Rightarrow \Delta S^d \downarrow \)
      (2) Y^e is unobservable. Thus we use proxies like
         (a) the index of consumer sentiment
         (b) consumer confidence index
   c. Wealth (stock, Assets-Liabilities)
      (1) \( \Delta W \uparrow \Rightarrow \Delta C^d \uparrow \Rightarrow \Delta S^d \downarrow \)
      (2) e.g. changes in the value of the stock market
d. Expected Real Interest Rate
   (1) Recall that \( r^e = R - \pi^e \), which is the rate on which the decision to borrow or lend is made (from chapter 2). The decision to borrow or lend is synonymous with the decision to consume or save, respectively.

   \[
   (2) \quad r^e \uparrow \Rightarrow \Delta C^d \downarrow \\
   \Rightarrow \Delta S^d \uparrow
   \]

   i.e. as the expected return on savings rises, it becomes cheaper to enjoy consumption in the future and more expensive to enjoy current consumption.

e. Taxes (taxes on interest income)
   (1) If we modify the expected real rate to include taxes paid on interest income, we have the after-tax expected real return,

   \[
   r^e_{at} = (1 - t)R - \pi^e
   \]

   where “\( t \)” is the marginal tax rate and \( 0 < t < 1 \).

   \[
   (2) \quad t \uparrow \Rightarrow r^e_{at} \downarrow \Rightarrow \Delta C^d \uparrow \\
   \Rightarrow \Delta S^d \downarrow
   \]

f. Fiscal Policy - Government Expenditures
   Assumption:
   \( \Delta G \uparrow \) does not affect \( K \) or \( L \) and therefore does not \( \Delta Y \uparrow \) (i.e. the economy is already at full employment)

   Then,
   (1) \( \Delta G \uparrow \) (financed by \( \Delta T \) \( \uparrow \)) \( \Rightarrow \Delta Y \downarrow \Rightarrow \Delta C^d \downarrow \\
   \Rightarrow \Delta S^d \downarrow
   
   (2) \( \Delta G \uparrow \) (financed by \( \Delta T \)-Bonds \( \uparrow \))
       \[\text{[Assuming that the public understands the government’s intertemporal budget constraint.]} \Rightarrow \Delta T^e \uparrow \Rightarrow Y^e \downarrow \Rightarrow \Delta C^d \downarrow \\
   \Rightarrow \Delta S^d = 0 \\
   (\Delta S_{prvt} \uparrow, \Delta S_{govt} \downarrow)\]

   (3) \( \Delta G \uparrow \) (financed by \( \Delta T \)-Bonds \( \uparrow \))
       \[\text{[Assuming that the public does not understand the government’s intertemporal budget constraint.]} \Rightarrow \Delta C^d = 0 \\
   \Rightarrow \Delta S^d \downarrow \\
   (\Delta S_{prvt} = 0, \Delta S_{govt} \downarrow)\]
g. Fiscal Policy - Taxes

Assumption: Tax Cut \((\Delta T \downarrow \text{ and } \Delta G = 0)\)

1. [Assuming that the public understands the government’s intertemporal budget constraint.]

\(\Delta T \downarrow \) and public anticipates \(\Delta T^* \uparrow \Rightarrow Y^* \downarrow \) and therefore,

\[\Rightarrow \Delta S_{priv} \uparrow \text{ by the amount of the tax cut} \]
\[\Rightarrow \Delta S_{govt} \downarrow \text{ by the amount of the tax cut} \]

\(\Rightarrow \Delta S \text{ remains constant and } \Delta C^d \text{ remains constant} \)

(i.e., the tax cut has no effect on the national economy)

(According to the Ricardian Equivalence Theorem, these two effects cancel such that the tax cut has not affect on \(C^d\) and increases \(S^d\) by the amount of the tax cut, which will be used to pay the future tax bill. The end effect is that \(C^d\) and \(S^d\) will remain unchanged.)

2. [Assuming that the public does not understand the government’s intertemporal budget constraint.]

\(\Delta T \downarrow \) and public anticipates nothing and spends tax cut (in reality they would spend part of the tax cut)

\[\Rightarrow \Delta S_{priv} = 0 \]
\[\Rightarrow \Delta S_{govt} \downarrow \text{ by the amount of the tax cut} \]

\(\Rightarrow \Delta S \downarrow \text{ and } \Delta C^d \uparrow \)

(i.e., the tax cut has the effect of increasing consumption)

Desired Investment Expenditures \((I^d)\)

1. Investment is a flow that produces a change in the stock of capital at the end of the period. This stock of capital produces a flow of output in subsequent periods. Thus, \(I\) accounts for major changes in future L-run economic growth.
2. \(I\) depends on expectations of capital’s productivity and future demand for Goods & Services.
3. \(I\) accounts for only about 15% of GDP
4. \(\Delta I\), however, often accounts for around 50% of \(\Delta GDP\) during recessions. Thus \(I\) is thought to play a major role in explaining business cycles.

Desired Capital Stock (Desired Investment) – Neoclassical Theory: Assumes that lenders (banks) and borrowers (firms) have the same information set on the investment opportunities and intentions of the borrower.

The Desired Capital Stock \((K^* \text{ - which will eventually lead to } I^d)\)

a. A firm’s optimal \(K\) depends on the benefits and costs of using \(K\) inputs relative to others.

b. Benefit: productivity of \(K\) (i.e. the output it can produce and, therefore at a given price, the revenue it can bring to the firm).

c. Cost: the price of capital (which includes the rental price and depreciation)

d. User Cost of \(K\)

\[P_k \equiv \text{real price of capital (nominal price deflated by the appropriate price index). Measured in real } $\] 

\[d \equiv \text{depreciation rate of } K \text{ (measured as a } \% \text{ of the original real value)}\]
$r^e \equiv$ expected real interest rate paid on either renting the K or on
borrowing the funds to buy the K (measured as an annual %).

User Cost (uc) = $r^e P_k + dP_k$
(expected interest payments) (% of K depreciated)

e. Expected MPK (MPK^e)

$MPK^e = \text{expected future MPK}$

f. Desired (Optimal) K Stock

(1) as long as,

$MPK^e > uc$ the firm should want to increase its capital stock.

(1) the firm will attain its optimal capital stock ($K^*$) where,

$MPK^e = uc$

(2) graphically,

\begin{center}
\begin{tikzpicture}
\draw[->] (-0.5,0) -- (5,0) node[below] {$K$};
\draw[->] (0,-1) -- (0,5) node[left] {$MPK^e, \text{ uc}$};
\draw (0,0) -- (4,4) node[midway, above left] {uc};
\draw (4,0) node[below] {$K^*$};
\draw (0,0) node[below] {$K^D$};
\end{tikzpicture}
\end{center}

\begin{itemize}
  \item [(g).] Factors that change $K^*$
    \begin{enumerate}
    \item $\Delta P_k$ (-)
    \item $\Delta r^e$ (-)
    \item $\Delta d$ (-)
    \item $\Delta MPK^e$ (+)
    \end{enumerate}
\end{itemize}

h. Taxes and $K^*$ - Output is taxed (actually $P$ times output.). Therefore the after-tax output is

$(1- \tau) \text{MPK}^e$, where $\tau$ is the marginal tax rate on output (corporate income tax) and

$0 < \tau < 1$. ($P(1- \tau) \text{MPK}^e$ is the after-tax revenue or income).
(1) Tax policies that alter $K^*$
   (a) depreciation allowances
   (b) investment tax credit
   (c) The effective tax rate ($\tau^*$) is that rate that incorporates all provisions of the tax code (so that we can compare the tax rate between countries, e.g.).
   (d) In summary,
   \[ \Delta K^*/\Delta \tau^* < 0 \]

Desired Capital Stock (Desired Investment) – Imperfect Capital Markets: Assumes asymmetric information problems between lenders (banks) and borrowers (firms).

Assumptions:
   a. The borrower (firm) has more information on investment opportunities and intentions. => This asymmetric information can lead to moral hazard behavior (i.e., engaging in behavior that will decrease the probability of the loan being repaid) by the firm that increases the probability that the firm will not be able to repay the loan, thereby increasing the default risk of the firm. The existence of asymmetric information problems also implies that the lender has to use real resources to monitor the behavior of the borrower to mitigate moral hazard behavior.
   b. The net worth of the firm can play a role in the decision to lend. Firm net worth can act as collateral, which can guarantee that the lender is repaid.
   c. The asymmetric information problem can be more concisely argued by assuming two types of capital investment expenditures. Firms make an investment in Hard $K$ (e.g., buildings and equipment) and Soft $K$ (maintenance and organization). Use of Soft $K$ can increase the productivity of Hard $K$. Hard $K$ expenditures and use is monitored at low cost. Soft capital expenditures and use is costly to monitor. Because of asymmetric information, a firm may skimp on soft $K$, thereby reducing the $MPK^c$ and decreasing the probability of the loan being repaid. Since the $MPK$ could vary partially due to good or bad luck, as well as due to skimping on soft $K$ by the firm, it is difficult for the lender to determine whether an adverse outcome of the firm is due to bad luck or skimping on soft $K$.
   Therefore, the lender will require collateral to guarantee the loan. If the loan exceeds the value of the collateral, the lender will have incentive to monitor the firm more intensely (at a cost, of course).

Example A: Net worth of the firm is used as collateral, where collateral equals the value of the optimal loan demanded (i.e., $W_0=K^*$). The lender will not bear excessive default risk and the lender need not monitor the firm. In this case, the cost of the loan is the same as that of the neoclassical case. The firm can achieve its optimal level of capital.

\[ MPK^c, uc \]
Example B: Net worth of the firm is used as collateral, where collateral is less than the value of the optimal loan demanded (i.e., $W_0 < K^*$). The lender will bear excessive default risk associated with the non-collateralized portion of the loan and, therefore, the lender has incentive to monitor the firm. In this case, the cost of the loan rises from the perfect information or fully collateralized cases. The cost of the loan rises as the amount of the loans rises above the collateralized amount (this is depicted by a rising supply of funds “$S$” in the figure below). This higher cost reflects greater default risk that the lender is exposed to and the higher costs associated with monitoring. The firm cannot achieve its optimal level of capital due to the higher cost of capital. Note that due to information problems some firms in the economy will not be able to achieve their optimal amount of capital and therefore the economy cannot benefit from the high yielding investment projects associated with these firms.

Factors that change $K^*$ in our two models of optimal Capital:

1. $\Delta P_k$ ($-$)
2. $\Delta r^e$ ($-$)
3. $\Delta d$ ($-$)
4. $\Delta MPKe$ ($+$) (This shifts $K^D$ and is made up of $\Delta$Labor quantity, $\Delta$Labor productivity and $\Delta$Technology)
5. $\Delta \tau^*$ ($-$)
6. Risk of firm default ($-$)
7. Cost of monitoring firms ($-$)
8. Net Worth available as collateral ($+$)

Imperfect Capital Markets and Volatility of Investment Expenditures:
If we view the economy as made up of two types of firms: those that are not affected by asymmetric information problems (e.g., large and well-established firms on which information is readily available, and small and new firms on which information is scarce). As the economy slows down the net worth of firms decreases and the risk of default increases. This situation may not drastically affect large and well-established firms, but it will decrease the available collateral of small firms and increase the cost of monitoring for this latter group. This will cause the capital stock of these firms to fall drastically and therefore cause investment expenditures to fall drastically at these firms. The opposite will occur during expansions. Thus, the argument of imperfect capital markets helps to explain the volatility of Investment Expenditures, which it attributes to small and new firms.
6. The Relationship between $K^*$ and $I^d$
   a. Gross Investment
   \[ I_t = K_{t+1} - K_t + d K_t \]
   b. Net Investment
   \[ I_{t, NET} = K_{t+1} - K_t = I_t - d K_t \]
   c. Investment in terms of a firm’s optimal $K^*$
      (1) Assumptions
         (a) a firm determines $K^*$ based on information in the previous period
         (b) a firm can achieve $K^*$ through optimal $I$ at the end of the next period. (not realistic for some $K$. There is usually a lag in building up $K$)
      Then,
      \[ I_{t}^d = K^* - K_t + d K_t \]
      Since $K_t$ is predetermined and $d$ is determined by technology, all the factors that affect $K^*$ also determine $I_t^d$

7. Goods Market Equilibrium ($S_t^d = I_t^d$ )
   a. Equilibrium Concept:
      Aggregate Supply of goods ($y$) = $Y^d = C^d + I^d + G$
      \[ Y - C^d - G = I^d \]
      The LHS is Desired National Savings, i.e.
      \[ S^d = I^d \]
   b. If $S^d > I^d$ then $C^d$ will not be high enough to consume all goods and services produced => inventory build-up (unintentional)
   c. If $S^d < I^d$ then $C^d$ will be too high and demand will be greater than goods and services produced => inventory reduction (unintentional)
d. Equilibrium Graphically

![Equilibrium Graphically Diagram]

e. Shifts in $S^d$  Shifts in $I^d$

(1) $\Delta Y$  (1) $\Delta P_k$
(2) $\Delta Y^c$  (2) $\Delta d$
(3) $\Delta G$  (3) $\Delta MPK^c$
(4) $\Delta \text{Wealth}$  (4) $\Delta \tau^*$
(5) $\Delta T$  (5) Risk of firm default
(6) $\Delta t$  (6) Cost of monitoring firms
(7) Net worth available as collateral

f. Movements along $S^d$  Movements along $I^d$

$\Delta r_{e,t}^c$  $\Delta r^e$

The above shocks affect equilibrium real interest rates ($r$) and equilibrium I and S.
Chapter 6  
Long-run Economic Growth

**Growth Accounting Equation** – explain the contribution of capital, labor and productivity on long-run economic growth.

\[ \Delta Y/Y = \Delta A/A + \alpha(\Delta K/K) + \beta(\Delta L/L) \]

Growth in TFP very important for countries to achieve a high sustainable growth rate.

Productivity Slowdown of 1973-1997 possibly due to 4 factors

1. Measurement Error
2. Legal/Human Environment
3. Technological Depletion/Commercial Adaptation
4. Oil price increases

**Shortcomings of the Growth Accounting Equation**

1. Relates K, Land TFP growth to Y growth, but doesn’t address what causes K, L and TFP to grow
2. Treats TFP growth as a residual. But there could be problems with measuring K and L
The Solow Growth Model

Variables and Definitions:

1. \( Y_t = \text{GDP=GDI} \)
2. \( L_t = \text{Labor} \)
3. \( C_t = \text{consumption expenditures} \)
4. \( S_t = \text{savings} \)
5. \( y_t = Y / L \) (per worker output)
6. \( c_t = C / L \) (per worker consumption)
7. \( k_t = K / L \) (capital-labor ratio)
8. \( s = \text{savings rate (% of income saved)} \)
9. \( l = \text{growth rate of the population (this is exogenous, not determined by policy or by firms)} \)
10. \( d = \text{depreciation rate of capital} \)

Production Function

\[
\begin{align*}
(1) & \quad Y_t = AF(K_t, L_t) \\
(2) & \quad \frac{Y_t}{L_t} = AF\left(\frac{K_t}{L_t}, 1\right), \quad \text{(per worker production function)} \\
& \quad y_t = f(k_t) \quad \text{(per worker production function in simpler notation)}
\end{align*}
\]

1. This production function still exhibits the same properties of our usual production function. Note, however, that when \( K \) and \( L \) grow at different rates (long-run or short-run) that \( f(k) \) exhibits diminishing marginal product in each of its inputs.

FIGURE 1

\( y_t \)

\( f(k_t) \)

\( k_t \)

i.e. as \( k \) rises (which means \( K_t \) grows faster than \( L_t \)), \( y_t \) increases at a decreasing rate (which means you have more output per worker, but additional increases in output per worker get smaller and smaller)
2. **The Steady State**

A steady state is a dynamic equilibrium. Instead of a “static equilibrium” where all variables are constant, in a steady state equilibrium all variables grow at constant rates. Thus, a steady state equilibrium will be one in which all variables per worker will grow at constant rates. Since population (and therefore $L_t$) grows at a constant rate “$l$,” then, $C_t$, $Y_t$, and $K_t$ must also grow at the same constant rate in the long-run to reach the steady state equilibrium. This means that $c_t$, $y_t$ and $k_t$ must be constant in the long-run. I.e. if the population (and the labor force) grows at rate “$l$” capital, consumption and output must also grow at the same rate to keep the above per worker variables constant.

But in the process of capital formation, some capital will wear out. So that to maintain enough “net capital” to grow at rate “$l$,” the gross capital stock must be growing faster. I.e. capital growth must take into account the growth rate of population and the depreciation rate of capital.

\[ \Delta K_t = lk_t + dk_t, \text{ or} \]

\[ \Delta (K_t/K_t) = l + d \]

i.e. the steady state growth rate of capital must equal the population growth rate plus the depreciation rate.

Recalling the definition of Investment and using eq. (3),

\[ I_t = (l + d)K_t \]

\[ i_t = (l + d)k_t \]

(investment per worker)

Recall that in equilibrium $S_t = I_t$. Likewise, in the steady state this will hold. Then,

\[ Y_t - C_t = I_t, \text{ or } C_t = Y_t - I_t. \]

Then, combining (5) and (7),

\[ C_t = Y_t - (l + d)K_t, \text{ or in per worker form, and substituting (2) in for } y \]

\[ c_t = f(k_t) - (l + d)k_t \]

eq. (9) tells us that consumption (per worker) is comprised of two parts

a. Income (or equivalently output)

b. Minus savings (which goes to finance the growth in net capital).

As income (and output) grows it adds to consumption. As savings (and therefore investment) grows it subtracts from consumption.

- Eq. (9) is consumption per worker in the steady state when capital grows at the steady state as in Eq. (4). This equation also shows us the relationship between consumption and the capital-labor ratio.
Steady state $k$, $y$, $c$ and $i$.

Using Eq. (9) we can graph the steady state level of all variables (i.e. eqs. (2) and (6) which form eq. (9)). $y$ depends on $k$, and $i$ depends on $k$, and therefore $c$ depends on $k$.

\[
i_t = (l+d) k_t
\]
\[
y_t = f(k_t)
\]

Our long-run goal for the economy is to maximize consumption per worker (or in terms of the population, per capita) in the long-run. We will consider this the maximization of our standard of living. Since consumption per worker is dependent on capital per worker, we wish to choose level of $k$ so as to maximize $c$.

$C_{\text{max}}$ is where output minus investment ($C_{\text{max}} = y - I$) is at a maximized. The figure above shows the value of $k$ ($k^*$) that maximizes consumption (the distance between $y$ and $i$).

Note: For you calculus buffs, if we take the derivative of $C$ wrt $k$ and set this equal to zero to maximize $C$ (yes, I know the $2^{\text{nd}}$ derivative must be $< 0$ for a maximum, and it is since the production function exhibits diminishing MPK),

\[
\Delta C / \Delta k = f'' - (d + l) = 0, \quad \text{and} \quad f'' < 0
\]

or

\[
f'' = d + l
\]

Thus, the maximum level of consumption is where the slope of the production function ($f''$, which is the MPK) is equal to the slope of $i$ (which is $d+l$).

If we try to maximize consumption in the steady state (i.e. in the long-run) we are providing the maximum amount of per capita consumption for all future generations. Thus, the term the Golden Rule level of consumption and capital stock is the amount we give to others as we would have them give to us.
Summary,
1. Long-run consumption per capita depends on output and savings
2. Output is determined by capital and labor
3. Capital is determined by investment
4. Since the population grows at an exogenously determined rate and the depreciation rate is also exogenous, capital must grow at the rate of the sum of these in order to maintain k.
5. Our Slow model says that an optimal k (k*) can be chosen to maximize per capita consumption. If we save too much and thus invest too much we will have a larger than optimal capital stock. Then we have to keep saving at a high rate in order to maintain this large capital stock. This takes away from consumption, even in the long-run. If we save too little and invest too little, we will not have a sufficient capital stock to obtain enough output to maximize our consumption.
6. The model, up to this point, has not mentioned whether we do reach this maximum consumption per worker naturally or how we reach this maximum. Also, it does not tell us whether we can increase our steady state per capita consumption over time.

Shocks and Re-equilibrating to the Steady State
1. Above we showed that in equilibrium $S_t = I_t$. But we did so by arguing that consumption was output minus savings. Savings was thus treated as a residual after consumption, without specifying what might determine savings and consumption. Now we will specify savings behavior. This will help us achieve an argument for reaching the steady state. As in Chapter 4, we will assume that savings is a constant fraction of income (Recall MPS?),

$$s_t = sY_t,$$

where $0 < s < 1$ is the savings rate. In per capita terms,

$$s_t = sY_t = sf(k_t),$$

which is the amount saved out of income (or output from eq. (2)).

Referring back to the steady state where $S = I$ (which we really expressed in terms of output and investment) and combining eqs. (6) and (12),

$$s_t = i_t,$$

or

$$sf(k_t) = (1 + d) k_t,$$

(This must hold in the steady state)

Now we can argue as to how this model reaches the steady state just as we argued this equilibrium between $S$ and $I$ in the static models of chapter 4.

Eq. (14) LHS shows that savings per worker depends on the capital-labor ratio and rises at a decreasing rate, reflecting diminishing returns to capital. The RHS implies that as the capital-labor ratio rises, investment must rise at rate $(1 + d)Ak_t$. The steady state and the adjustment to this state can be easily seen in the following figure,
where $k^*$ depicts the capital-labor ratio in which $s_t = i_t$. Equilibrium is achieved by the following argument: If $k > k^*$, then $s_t < i_t$, savings will not be sufficient to maintain the investment needed to maintain $k_2$. Hence, investment will fall and $k$ will fall. Likewise, if $k < k^*$, then $s_t > i_t$ and savings will be greater than the investment needed to maintain $k_1$. Hence investment will rise and $k$ will rise.

Note that the amount of savings drives us to equilibrium. I.e. the assumption on how savings is formed "buys" us the equilibrium. Investment will rise or fall to equate to savings. Without the savings at $k_2$ investment cannot maintain this level of capital. With greater savings at $k_1$ investment will increase the stock of capital.

At our long-run steady state of capital ($k^*$), we can compute steady state $y$ and $c$. Plugging $k^*$ into eqs. (2) and (9) we obtain,

\begin{align}
(15) \quad y_t^* &= f(k_t^*) \\
(16) \quad c_t^* &= f(k_t^*) - (l + d)k_t^*
\end{align}

where all variables grow at rate $l$ ($L, Y, C, K^{\text{NET}}, I^{\text{NET}}$). This result implies that our standard of living will remain constant over time. Next we explain how we can increase our standard of living.

**Determinants of the Long-Run Standard of Living**

1. Increase in the Savings Rate: If the savings rate grows $sf(k)$ will shift up and a higher level of output and consumption and capital per worker in the long-run. But the increase in savings leads to falling consumption per worker in the short-run.
2. Increase in Population: An increase in the population causes the $(l+d)k$ line to become steeper. Thus capital must grow faster to maintain the same capital-labor ratio. If it does not (due to other factors that would cause it to do so) the capital-labor ratio falls and output and consumption fall also.
3. An Increase in Productivity Growth: An increase in productivity growth shifts the production function up (i.e. \( f(k) \) rises) and thus savings (\( sf(k) \)) increases. The productivity increase raises output per worker. It also raises the capital-labor ratio and consumption per worker. This is similar to an increase in the savings rate. This raises output and consumption in two ways. It increases the amount that can be produced for any \( k \). It also increases the long-run capital-labor ratio. This also increases output and consumption.

Note: With all these shocks that increase per worker consumption and output, growth initially increases. Once the steady state is reached the output per worker and consumption per worker become constant.