GSB 420
Homework #2b Calculus Applications: Optimization

A. Find the value of X that optimizes the following functions. Identify whether X value found is associated with a maximum or a minimum of the primary function, Y. find the value of Y at the optimal value of X

1. $Y = 2X^2 - 4X + 20$

   a. X=1; Minimum; Y=18
   b. X=1; Maximum; Y=18
   c. X=2; Minimum; Y=20
   d. X=2; Maximum; Y=20
   e. none of the above

2. $Y = -4X^2 + 16X + 1$

   a. X=1; Minimum; Y=13
   b. X=1; Maximum; Y=13
   c. X=2; Minimum; Y=17
   d. X=2; Maximum; Y=17
   e. none of the above

3. $Y = -2(X - 5)^2$

   a. X=5; Minimum; Y=0
   b. X=5; Maximum; Y=0
   c. X=-5; Minimum; Y=200
   d. X=-5; Maximum; Y=200
   e. none of the above

4. $Y = \frac{1}{3}X^3 - \frac{3}{2}X^2 + 2X + 1$

   a. X=1; Minimum; Y=11/6
   b. X=1; Maximum; Y=11/6
   c. X=2; Minimum; Y=11/6
   d. X=2; Maximum; Y=5/3
   e. none of the above

5. $Y = \frac{20}{3}X^3 - 60X^2 + 160X + 10$

a. X=2; Minimum; Y=143.333
b. X=2; Maximum; Y=143.333
c. X=4; Minimum; Y=116.666
d. X=4; Maximum; Y=116.666
e. only (b) and (c) of the above

B. Find the first and the second partial derivatives of the following multivariate functions with respect to X.

6. $Y = 2X^3Z^5$

a. $\frac{\partial Y}{\partial X} = 6X^2Z^5$ $\frac{\partial^2 Y}{\partial X^2} = 12XZ^5$

b. $\frac{\partial Y}{\partial X} = 10X^3Z^4$ $\frac{\partial^2 Y}{\partial X^2} = 40X^3Z^3$

c. $\frac{\partial Y}{\partial X} = 3X^2Z^5$ $\frac{\partial^2 Y}{\partial X^2} = 6XZ^5$

d. $\frac{\partial Y}{\partial X} = 6X^3Z^5$ $\frac{\partial^2 Y}{\partial X^2} = 18X^3Z^5$

e. none of the above

7. $Y = 10X^aZ^b - 2X + 3Z$

a. $\frac{\partial Y}{\partial X} = 10aX^{a-1}Z^{b-1} - 2$ $\frac{\partial^2 Y}{\partial X^2} = 10a(a-1)X^{a-2}Z^{b-2}$

b. $\frac{\partial Y}{\partial X} = 10bX^aZ^{b-1} + 3$ $\frac{\partial^2 Y}{\partial X^2} = 10b(b-1)X^aZ^{b-2}$

c. $\frac{\partial Y}{\partial X} = 10aX^{a-1}Z^b - 2$ $\frac{\partial^2 Y}{\partial X^2} = 10a(a-1)X^{a-2}Z^b$

d. $\frac{\partial Y}{\partial X} = 10aX^aZ^b - 2$ $\frac{\partial^2 Y}{\partial x^2} = 10a(a-1)X^{a-1}Z^b$

e. none of the above

8. $Y = 2X^{-3}Z^2 + 5X^2Z - 4X + 20Z$

a. $\frac{\partial Y}{\partial X} = 6X^{-4}Z^2 + 10XZ - 4$ $\quad\quad$ $\frac{\partial^2 Y}{\partial X^2} = -24X^{-5}Z^2 + 10Z$

b. $\frac{\partial Y}{\partial X} = 4X^{-3}Z \ + 5X^2 + 20$ $\quad\quad$ $\frac{\partial^2 Y}{\partial X^2} = 4X^{-3}$

c. $\frac{\partial Y}{\partial X} = -6X^{-4}Z^2 + 10XZ - 4$ $\quad\quad$ $\frac{\partial^2 Y}{\partial X^2} = 24X^{-5}Z^2 + 10Z$

d. $\frac{\partial Y}{\partial X} = -6X^{-2}Z^2 + 10XZ - 4$ $\quad\quad$ $\frac{\partial^2 Y}{\partial X^2} = 12X^{-1}Z^2 + 10Z$

e. none of the above

9. $Y = \frac{X^2}{Z^2} + 3X^5 - 2Z^3 + XZ$

a. $\frac{\partial Y}{\partial X} = \frac{2X}{Z^2} + 15X^6 + Z$ $\quad\quad$ $\frac{\partial^2 Y}{\partial X^2} = \frac{2}{Z^2} + 90X^7$

b. $\frac{\partial Y}{\partial X} = \frac{2}{Z^2}X + 15X^4 + Z$ $\quad\quad$ $\frac{\partial^2 Y}{\partial X^2} = \frac{2}{Z^2} + 60X^3$

c. $\frac{\partial Y}{\partial X} = \frac{2}{Z^2}X + 15X^4$ $\quad\quad$ $\frac{\partial^2 Y}{\partial X^2} = \frac{2}{Z^2} + 60X^3$

d. $\frac{\partial Y}{\partial X} = \frac{X}{2Z^2} + 15X^4 + Z$ $\quad\quad$ $\frac{\partial^2 Y}{\partial X^2} = \frac{1}{2Z^2} + 60X^3$

e. none of the above

10. $Y = 5X^{10} + 10Z^3 + 2X^{-2}Z^{-3} + 20X - 10Z + 100$

a. $\frac{\partial Y}{\partial X} = 30Z^2 - 6X^{-2}Z^{-4} - 10$ $\quad\quad$ $\frac{\partial^2 Y}{\partial X^2} = 60Z + 24X^{-2}Z^{-5}$

b. $\frac{\partial Y}{\partial X} = 50X^9 - 4X^{-3}Z^{-3} + 20$ $\quad\quad$ $\frac{\partial^2 Y}{\partial X^2} = 450X^8 + 12X^{-4}Z^{-3}$

c. $\frac{\partial Y}{\partial X} = 50X^9 + 10Z^2 + 20 - 10Z$ $\frac{\partial^2 Y}{\partial X^2} = 450X^8 + 10Z - 10$

d. $\frac{\partial Y}{\partial X} = 50X^9 + 30Z^2 - 4X^{-3}Z^{-3} + 20 - 10$

$\frac{\partial^2 Y}{\partial X^2} = 450X^8 + 60Z + 12X^{-4}Z^{-3}$

e. only (a) and (b) of the above

C. Find the first and the second partial derivatives of the following multivariate functions with respect to Z.

11. $Y = 2X^3Z^5$

a. $\frac{\partial Y}{\partial Z} = 10X^3Z^4$ $\frac{\partial^2 Y}{\partial Z^2} = 40X^3Z^3$

b. $\frac{\partial Y}{\partial Z} = 2X^3Z^4$ $\frac{\partial^2 Y}{\partial Z^2} = 8X^3Z^3$

c. $\frac{\partial Y}{\partial Z} = 4 \cdot 3 \cdot 2X^3Z^4$ $\frac{\partial^2 Y}{\partial Z^2} = 3 \cdot 24X^3Z^3$

d. $\frac{\partial Y}{\partial Z} = 30X^2Z^4$ $\frac{\partial^2 Y}{\partial Z^2} = 240X^1Z^3$

e. none of the above

12. $Y = 10X^aZ^b - 2X + 3Z$

a. $\frac{\partial Y}{\partial Z} = 10abX^{a-1}Z^{b-1} + 3$ $\frac{\partial^2 Y}{\partial Z^2} = 10ab(a-1)(b-1)X^{a-2}Z^{b-2}$

b. $\frac{\partial Y}{\partial Z} = 10bX^aZ^{b-1} - 2X + 3$ $\frac{\partial^2 Y}{\partial Z^2} = 10b(b-1)X^aZ^{b-2} - 2X$

c. $\frac{\partial Y}{\partial Z} = 3$ $\frac{\partial^2 Y}{\partial Z^2} = 0$

d. $\frac{\partial Y}{\partial Z} = 10bX^{a}Z^{b-1} + 3$ $\frac{\partial^2 Y}{\partial Z^2} = 10b(b-1)X^{a}Z^{b-2}$

e. none of the above

13. $Y = 2X^{-3}Z^2 + 5X^2Z - 4X + 20Z$

a. $\frac{\partial Y}{\partial Z} = 4X^{-3}Z \ + 20$ $\frac{\partial^2 Y}{\partial Z^2} = 4X^{-3}$

b. $\frac{\partial Y}{\partial Z} = 4X^{-3}Z \ + 5X^2 + 20$ $\frac{\partial^2 Y}{\partial Z^2} = 4X^{-3}$

c. $\frac{\partial Y}{\partial Z} = -6X^{-2}Z \ + 5X^2 + 20$ $\frac{\partial^2 Y}{\partial Z^2} = 12X^{-3}$

d. $\frac{\partial Y}{\partial Z} = 4X^{-3}Z \ + 5X^2 + 20$ $\frac{\partial^2 Y}{\partial Z^2} = 4X^{-3} + 10X$

e. none of the above

14. $Y = \frac{X^2}{Z^2} + 3X^5 - 2Z^3 + XZ$

a. $\frac{\partial Y}{\partial Z} = \frac{-2X^2Z}{Z^4} - 6Z^2 + X$ $\frac{\partial^2 Y}{\partial Z^2} = \frac{6X^2Z^4}{Z^8} - 12Z$

b. $\frac{\partial Y}{\partial Z} = \frac{-2X^2}{Z^3} - 6Z^2 + X$ $\frac{\partial^2 Y}{\partial Z^2} = \frac{6X^2}{Z^4} - 12Z$

c. $\frac{\partial Y}{\partial Z} = \frac{\partial^2 Y}{\partial Z^2}$

d. $\frac{\partial Y}{\partial Z} = \frac{2X^2}{Z^1} - 6Z^2 + X$ $\frac{\partial^2 Y}{\partial Z^2} = \frac{2X^2}{Z^0} - 12Z$

e. only (a) and (b) of the above

15. $Y = 5X^{10} + 10Z^3 + 2X^{-2}Z^{-3} + 20X - 10Z + 100$

a. $\frac{\partial Y}{\partial Z} = 50X^9 - 4X^{-3}Z^{-3} + 20$ $\quad$ $\frac{\partial^2 Y}{\partial Z^2} = 450X^8 + 12X^{-4}Z^{-3}$

b. $\frac{\partial Y}{\partial Z} = 30Z^4 - 6X^{-2}Z^{-2} - 10$ $\quad$ $\frac{\partial^2 Y}{\partial Z^2} = 120Z^5 + 12X^{-2}Z^{-1}$

c. $\frac{\partial Y}{\partial Z} = 30Z^2 - 6X^{-2}Z^{-4} - 10$ $\quad$ $\frac{\partial^2 Y}{\partial Z^2} = 60Z + 24X^{-2}Z^{-5}$

d. $\frac{\partial Y}{\partial Z} = 50X^9 + 30Z^2 - 4X^{-3}Z^{-3} - 10$ $\quad$ $\frac{\partial^2 Y}{\partial Z^2} = 60Z + 12X^{-4}Z^{-2}$

e. none of the above

D. Solve the following constrained optimization problems:

16. The following is the Sales (S) equation based on two types of advertising expenditures – newspaper advertising expenditure (X) and magazine advertising expenditure (Y):

$$S = 200X + 100Y - 10X^2 - 20Y^2 + 20XY$$

Assuming the total advertising budget is restricted to 20 (i.e., X+Y=20), find X and Y that maximize the sales (S), and identify the value of maximum sales (S) and the corresponding λ value. **(2 points)**

a. X=7; Y=13; S=2450; λ=80

b. X=7; Y=13; S=650; λ=80

c. X=13; Y=7; S=650; λ=80

d. X=13; Y=7; S=2450; λ=80

e. none of the above

17. Find the values of X and Y that maximize the following function and a constraint: (Do not solve for the second order conditions.) **(3 points)**

Maximize $S = -60 + 140X + 100Y - 10X^2 - 8Y^2 - 6XY$
Subject to: $20X + 40Y = 200$

Along with X and Y that maximize the sales (S), identify the value of maximum sales (S) and the corresponding λ value.

a. X=5.56; Y=2.22; S=837.97; λ=0.77

b. X=5.56; Y=2.22; S=517.62; λ=0.77

c. X=5.56; Y=2.22; S=517.62; λ=31.08

d. X=2.22; Y=5.56; S=436.15; λ=0.77

e. X=2.22; Y=5.56; S=436.15; λ=31.08