# GSB 420
## Homework #2c Calculus Applications: Optimization & Elasticity

A. Calculation of Demand Elasticity

Suppose the follow demand relationship between the number of automobiles (Q) and its per-unit price (P) is identified by your analyst:

$$Q = 300{,}000 - 20P$$

1. How many automobiles would be demanded at a price of $2,000?

   a. 40,000
   b. 80,000
   c. 120,000
   d. 260,000
   e. 300,000

2. The number of automobiles demanded ($Q_1$) at a price of $4,000 is _____, and the same ($Q_2$) at a price of $6,000 is _____.

   a. $Q_1 = 220{,}000$; and $Q_2 = 180{,}000$
   b. $Q_1 = 180{,}000$; and $Q_2 = 220{,}000$
   c. $Q_1 = 220{,}000$; and $Q_2 = 220{,}000$
   d. $Q_1 = 180{,}000$; and $Q_2 = 180{,}000$
   e. none of the above

3. Given the equation for the point own price elasticity of demand as:

$$|\varepsilon_d| = \frac{dQ}{dP} \cdot \frac{P}{Q}$$

   Calculate the point own price elasticity of demand at P=$2,000.

   a. -20 → 20
   b. -0.1538 → 0.1538
   c. -130 → 130
   d. -6.5 → 6.5
   e. not calculable

4. Given the equation for the point own price elasticity of demand as in Problem 3 above, calculate the point own price elasticity of demand at P=$4,000.

a. -20 → 20
b. -0.3636 → 0.3636
c. -55 → 55
d. -2.75→ 2.75
e. not calculable

5. Given the arc own price elasticity of demand as:

$$\left|\varepsilon_a\right| = \frac{dQ}{dP} \cdot \frac{\overline{P}}{\overline{Q}}$$

where $\overline{P}$ = average of two prices

and $\overline{Q}$ = average of two quantities

Calculate the arc own price elasticity of demand between P=$2,000 and P=$4,000.

a. -0.3636 → 0.3636
b. -0.1538 → 0.1538
c. -0.25 → 0.25
d. -4 → 4
e. not calculable

6. Given the equation for the arc own price elasticity of demand as in Problem 5 above, calculate the arc own price elasticity of demand between P=$4,000 and P=$6,000.

a. -0.3636 → 0.3636
b. -0.6667 → 0.6667
c. -0.5 → 0.5
d. -2 → 2
e. not calculable

7. If 200,000 automobiles were demanded and sold last year, what was the (per-unit) price of the automobiles?

a. $20,000
b. $15,000
c. $10,000
d. $5,000
e. not calculable

B. Advanced Problems in Demand Elasticity - 1

Given the following demand equation for Love Chocolate Bars (Q),

$$Q = 10 - 5P + 2I - 3P_c$$

where P = the price of a Love Chocolate Bar; I = average disposable income of the consumers ($) for chocolate bars; and $P_c$ = the price of a competitor's chocolate bar

Given a generic elasticity of demand as:

$$\varepsilon_X = \frac{\partial Q}{\partial X} \cdot \frac{X}{Q}$$

8. The point own price elasticity of demand at P=$2, given I = $20 and $P_c$ = 1, is equal to _____.

   a. -0.27 → 0.27
   b. -0.6667 → 0.6667
   c. -3.7 → 3.7
   d. -2.7 → 2.7
   e. none of the above

9. The point income elasticity of demand at I = $10, given P=$2 and $P_c$ = 1, is equal to _____.

   a. -1.176
   b. 1.176
   c. -0.85
   d. 0.85
   e. none of the above

10. The point cross-price elasticity of demand at $P_c$ = 2, given I = $10 and P=$3, is equal to _____.

   a. -3
   b. -1.5
   c. -0.667
   d. 0.667
   e. none of the above

C. Advanced Problems in Demand Elasticity - 2

Given the following demand equation for Love chocolate bars (Q),

$$Q = 10P^{-2}I^{3}A^{-4}P_c$$

where P = the price of a Love chocolate bar; I = average disposable income of the consumers ($) for chocolate bars; A= advertising expense for Love chocolate bars; and $P_c$ = the price of a competitor's chocolate bar

Given a generic elasticity of demand as:

$$\varepsilon_X = \frac{\partial Q}{\partial X} \cdot \frac{X}{Q}$$

11. The point own price elasticity of demand is _____ and the point income elasticity of demand is _____.

    a. -0.2 → 0.2; -3
    b. -0.5 → 0.5; 3
    c. -3.7 → 3.7; -3
    d. -2 → 2; 3
    e. none of the above

12. The point advertising elasticity of demand is _____ and the point cross-price elasticity of demand is _____.

    a. 4; -1
    b. -4; -1
    c. 4; 1
    d. -4; 1
    e. none of the above

D. Optimization Techniques for Profit Maximization - 1

If a firm sells its product at a fixed price of $121 per unit and has the following total cost function,

$$TC = 0.02Q^3 - 3Q^2 + 175Q + 500$$

13. The profit-maximizing output, Q, is _____ and the corresponding maximum profit level is _____. **(2 points)**

    a. 10; -760
    b. 100; 7600
    c. 90; 4360
    d. 90; -4360
    e. none of the above

14. The marginal revenue (MR) function is _____ and the marginal cost (MC) function is _____.

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a. $MR = 121; MC = 0.06Q^2 - 6Q + 175$

b. $MR = 121Q; MC = 0.06Q^2 - 6Q$

c. $MR = 121; MC = 0.02Q^3 - 3Q^2 + 175Q + 500$

d. $MR = 121Q; MC = 0.02Q^3 - 3Q^2 + 175Q$

e. none of the above

15. By using the fundamental principle of profit maximizing, the profit maximizing output level, Q, is found to be _____ and the corresponding maximum profit is _____.

a. 10; -760
b. 100; 7600
c. 90; 4360
d. 90; -4360
e. none of the above

E. Optimization Techniques for Profit Maximization – 2

Given the following demand function,

$$Q = 300 - 3P$$

and total cost function,

$$TC = \frac{1}{600}Q^3 - \frac{1}{3}Q^2 + 50Q + \frac{1000}{3}$$

16. The profit-maximizing output, Q, is _____ and the maximum profit level is _____. **(3 points)**

a. 10; -165
b. 10; 165
c. 30; 1121.67
d. 100; 300
e. none of the above

17. The marginal revenue function is _____ and the marginal cost function is _____.

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a. $MR = 3; MC = \frac{1}{200}Q^2 - \frac{2}{3}Q + 50$

b. $MR = 100 - \frac{2}{3}Q; MC = \frac{1}{200}Q^2 - \frac{2}{3}Q + 50$

c. $MR = -\frac{2}{3}$; $MC = \frac{1}{200}Q^2 - \frac{2}{3}Q + 50$

d. $MR = 100 - \frac{2}{3}Q$; $MC = 0.005Q^2 - 6Q + 50$

e. none of the above

18. By using the fundamental principle of profit maximizing, the profit maximizing output level, Q, is found to be _____ and the corresponding maximum profit is _____.

a. 10; -165
b. 10; 165
c. 30; 1121.67
d. 100; 300
e. none of the above

F. Optimization Techniques for Profit Maximization – 3

19. A firm has a fixed cost of $5000 and per unit cost of production of $2. Also, the firm has the following demand function,

Demand: $Q = 10{,}000 - 1{,}000P$

This firm's total revenue (TR) function is _____ and total cost (TC) function is _____.

a. $TR = 10Q - \frac{Q^2}{1000}$; $TC = 2Q$

b. $TR = 10Q - \frac{Q^2}{1000}$; $TC = 5000$

c. $TR = 10Q - \frac{Q^2}{1000}$; $TC = 2Q + 5000$

d. $TR = 10{,}000Q - \frac{Q^2}{1000}$; $TC = 2Q + 5000$

e. none of the above

20. The profit-maximizing output, Q, is _____ and the maximum profit level is _____. **(3 points)**

a. 400; -$1,960
b. 4000; +$11,000
c. 400; +$24,000
d. 4000; +$13,000
e. none of the above